REFERENCE: **SYLLABUS ITEM 92** **WEBNOTES 323/314**

<u>Mathematical Derivation of the Keynesian Multiplier</u>

A certain fraction of a household´s income tends to be consumer further. This fraction is denoted by the Marginal Propensity to Consume (MPC). This fraction (a value below 1, which is the total income) is spent and re-spent on goods, creating an infinite geometric series, whose sum is a product of the Keynesian Multiplier. Mathematically, this series can be expressed as follows:

We start with an income x, presumably subsidized by the government. A fraction of this income is used to buy goods, generating a certain amount of income for the seller. The seller spends a fraction of this extra income on further goods and so on.

$$x, MPC \cdot x, MPC(MPC \cdot x), \ldots$$

$$\therefore x, xMPC, xMPC^2, xMPC^3$$

The general form of this infinite series is denoted by:

$$u_n = x(MPC)^n; where\ n \geq 0$$

Our goal is to find out what the overall extra income for the entire economy is as a result of this Keynesian multiplication. Therefore, we calculate the sum of the infinite geometric sequence which converges at a certain value.

$$S_{Keynes} \equiv \sum_{n=0}^{\infty} x(MPC)^n = \frac{u_1}{1-r}; where\ r\ is\ the\ common\ ratio$$

$$= \frac{x(MPC)^0}{(1-MPC)}$$

$$= \frac{x}{(1-MPC)}$$

This final formula gives us a tool to calculate the amount by which the income in an economy has been augmented. The same process of derivation can be achieved in terms of the Marginal Propensity to Withdraw (MPW), or rather the fraction of money that is **not** invested in goods and services. Therefore, we realize that:

$$MPC = 1 - MPW$$

Thus,

$$S_{Keynes} \equiv \sum_{n=0}^{\infty} x(1-MPW)^n$$

$$= \frac{x(1-MPW)^0}{1-(1-MPW)}$$

$$= \frac{x}{MPW}$$

It is furthermore important to realize that MPW is the addition of MPS + MPT + MPM, which allows the solving of any Keynesian problem on HL Paper 3 given any data.